1. Classical conventional wage model, log utility, no factor substitution.

   Endogenous variables: $w, v, g_K + \delta, c$.

$$w = \bar{w} \tag{W}$$

$$w = x - vk, \text{ or } w = x\left(1 - \frac{v}{\rho}\right) \tag{WP}$$

$$g_K + \delta = v - \beta \tag{G}$$

$$c = x - (g_K + \delta)k, \text{ or } c = x\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{CG}$$

   The conventional wage determines the real wage in the economy. Given the conventional wage, and the values for capital intensity and labor productivity, the profit rate is determined from (WP). Given the profit rate, the Cambridge equation (G) determines the growth rate, using which we obtain social consumption per worker c from the consumption–growth schedule (CG).

2. Neoclassical model, no technical change, no class distinction, Cobb-Douglas production function, log utility.

   Endogenous variables: $w_{ss}, v_{ss}, c_{ss}, k_{ss}$; $g_K = 0$. Note that $x = k^\alpha$, $v = \alpha k^{\alpha-1}$ (because of marginal productivity pricing). The first equation ensures full employment of labor if the real wage is free to vary.

$$w_{ss} = k_{ss}^\alpha - \alpha k_{ss}^{\alpha-1} k_{ss} = (1-\alpha)k_{ss}^\alpha \tag{$\bar{L}$}$$

$$v_{ss} = \alpha k_{ss}^{\alpha-1} \tag{v_{ss}}$$

$$c_{ss} = k_{ss}^\alpha - (\delta + n)k_{ss} \tag{c_{ss}}$$

$$k_{ss} = \left(\frac{\alpha}{\delta + \beta}\right)^{\frac{1}{1-\alpha}} \tag{k_{ss}}$$

   Everything is technologically determined. Equation (k_{ss}) solves for the steady state value of capital stock by setting zero consumption growth in the Euler equation $\dot{c}/c = \alpha k^{\alpha-1} - (\delta+\beta)$. The steady state capital intensity also pins down income distribution: on the one hand, the profit rate will be determined by (v_{ss}); on the other hand, the real wage will follow from the marginal productivity condition ($\bar{L}$), which also ensures full employment. The reason why full employment holds is that at each moment in time the capital intensity is constant, which means that aggregate capital stock grows at the same rate of the labor force: through changes in the real wage —which in turn happen because of a change in the technique in use— a Neoclassical economy is able to accommodate any increase in the labor force in order to ensure the constancy of k_{ss}.

3. As an exercise, write down the growth and distribution system arising in a conventional wage share economy, and saving (investment) made only by capitalists, but with choice of technique (that is, a Cobb–Douglas production function). This model would be close to Lewis' (1954) model of the dual economy.

4. Classical conventional wage share model, pure labor saving technical change

   Suppose that labor productivity grows exogenously at a rate $\gamma > 0$. This means that at time t, $x(t) = x(0)e^{\gamma t}$. Suppose further (in line with stylized facts about economic growth) that the output capital ratio ρ remains constant over time. The capital intensity at time t, $k(t)$ fulfills:

$$k(t) = \frac{K(t)}{N(t)} = \frac{K(t)}{X(t)}\frac{X(t)}{N(t)} = \frac{1}{\rho}x(t) = \frac{1}{\rho}x(0)e^{\gamma t} = k(0)e^{\gamma t}.$$

   The presence of technical change requires to modify the conventional wage model to a *conventional wage share* version, where the real wage grows at the same rate of the rate of growth of labor productivity.

   Suppose we fix the share of profits at $\bar{\pi} \equiv vK/X$. We have $\frac{w(t)}{x(t)} = 1 - \pi$ which implies $w(t) = (1-\bar{\pi})x(0)e^{\gamma t} = w(0)e^{\gamma}t$: the real wage also grows at the rate γ.

   From the real wage–profit share relation

$$\begin{aligned} w(t) &= x(t) - vk(t) \\ w(0)e^{\gamma t} &= x(0)e^{\gamma t} - vk(0)e^{\gamma t} \\ w(0) &= x(0) - vk(0) \end{aligned}$$

   which is basically the WP relation at time zero. The observation suggests to rescale variables so that they remain constant over time. Denote by $\tilde{w} = w(t)e^{-\gamma t}, \tilde{x} = x(t)e^{-\gamma t}, \tilde{k} = k(t)e^{-\gamma t}, \tilde{c} = c(t)e^{-\gamma t}$: all of these variables are constant. Thus, the conventional wage share model will have exactly the same structure of the conventional wage model, with the difference that we are considering "$\sim$" variables.

   Endogenous variables: $\tilde{w}, v, g_k + \delta, \tilde{c}$.

$$\tilde{w} = (1-\bar{\pi})\tilde{x} \tag{WS}$$

$$\tilde{w} = \tilde{x} - v\tilde{k}, \text{ or } \tilde{w} = \tilde{x}\left(1 - \frac{v}{\rho}\right) \tag{WP.1}$$

$$g_K + \delta = v - \beta \tag{G.1}$$

$$\tilde{c} = \tilde{x} - (g_K+\delta)\tilde{k}, \text{ or } \tilde{c} = \tilde{x}\left(1 - \frac{g_K+\delta}{\rho}\right) \tag{CG.1}$$

5. As an exercise, consider the choice of technique in the Classical model with a conventional wage share. Show that the intensive production function in units of effective labor is constant, and write the growth and distribution system for the model.

6. Classical full employment model.

   It is also interesting to consider a Classical model with a full employment closure, as opposed to a conventional wage share. Now, this is quite different from the Neoclassical case, because here the technique of production does not change with the real wage. Suppose that labor supply grows at the exogenous rate n, and that γ always denotes the growth rate of labor productivity. The effective labor supply, xN, grows at a rate equal to $n+\gamma$. Such rate is called the *natural* rate of growth. Suppose also that capital stock depreciates at the constant rate $\delta > 0$.

   The basic change to the model is the description of the labor market. In order to maintain full employment, capital accumulation has to match the increase in the labor supply every period. In other words, the growth rate of capital stock has to be equal to the natural rate of growth. This equation closes the model. We have:

$$g_k = \gamma + n \tag{$\bar{G}$}$$

$$\tilde{w} = \tilde{x} - v\tilde{k}, \text{ or } \tilde{w} = \tilde{x}\left(1 - \frac{v}{\rho}\right) \tag{WP.2}$$

$$g_K + \delta = v - \beta \tag{G.2}$$

$$\tilde{c} = \tilde{x} - (g_K + \delta)\tilde{k}, \text{ or } \tilde{c} = \tilde{x}\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{CG.1}$$

   Note the different type of recursion relative to the conventional wage share model. Equation ($\bar{G}$) pins down the growth rate given labor force growth and labor productivity growth. The growth rate determines the profit rate through (G.2), as well as consumption through (CG.1). After the profit rate is found, the wage share equation determines $\tilde{w}$.