1 The Goodwin (1967) Model

In 1967, Richard Goodwin developed an elegant model meant to describe the evolution of distributional conflict in growing, advanced capitalist economies. The Goodwin model is important because it tells a story about the dynamics of the growth and distribution process at the heart of the Foley and Michl approach.

The key variables in the Goodwin model are the share of labor in national income, and the employment rate. In the conventional wage share model of Foley and Michl, the labor share is always constant at its conventional level. While this is a reasonable approximation over the very long–run, the labor share fluctuates in actual economies. Similarly, in the conventional wage share model the amount of labor hired each period is given by $\rho K/x$, where ρ is the output/capital ratio, x is labor productivity. The dynamics of employment are 'in the back of the picture' in the conventional wage share model.

The main thrust of the Goodwin model is that distributional conflict produces *endogenous cycles* in employment and the labor share, and the key element through which this happens is the fact that out of a balanced growth path —a path where all variables grow at the same rate, namely the growth rate of labor productivity in the conventional wage share model— wages and labor productivity might grow at different rates. In particular, upward or downward pressure on wages may result from *tightness* in the labor market. The employment rate can be used as a measure of labor market tightness.

The economic intuition is the following. Suppose the economy is expanding, and employment increases. Higher labor demand generates wage inflation which, as long as real wages increase more than labor productivity, increases the wage share in output. If, consistently with the Classical view, workers do not save, the resulting decrease in the profit share will act in reducing future investment and output. But then the economy is down, and lower labor demand will then correspond to lower output, leading the way to lower wage inflation or even wage deflation. The labor share will decrease. But a higher profit share will produce a surge in investment, which will generate higher employment, thus improving workers' bargaining power and consequently wages. At this point, the wage share has increased, and the cycle can repeat itself.

Goodwin realized that this type of dynamics is also found in simple biological systems. Suppose that in a certain territory there is a predator species and a prey species. If predators are too little in number, preys will proliferate, thus pushing against the resource constraint in the territory. Prey proliferation, however, makes predators' life easier: they will be better fed, and reproduce at higher rate. But when the number of predators is too high, finding preys to eat becomes harder, and mortality among predators will increase. Preys will have more opportunities to reproduce, and the cycle can repeat itself.

In what follows, we will develop the Goodwin (1967) model, and discuss its predator-prey dynamics.

1.1 The Model

We consider a small modification of the original Goodwin (1967) model. The economy is populated by workers and capitalists. Workers supply labor services to firms owned by capitalists, do not save, and consume all of their income. Capitalists own capital assets, consume and save. Denote aggregate output by Y, capital by K and labor by L. Labor is homogeneous, hence we can use a single real wage rate, w. Production of output occurs with the following fixed-proportions technology:

$$Y = \min\{\rho K, xL\}$$

implying that real unit labor costs, that is the labor share, is $\omega \equiv w/x$. Labor productivity grows at the exogenous rate $\gamma > 0$, while the output/capital ratio ρ remains constant over time. We also assume exogenous growth of the labor force, at a rate $n > 0$. Further, we suppose that workers are willing to supply any amount of labor at the going real wage, so that labor supply is horizontal. Employment depends on firms' choices, then. Since full employment might or might not be attained, we denote the employment rate by $e \equiv L/N$, where N is the total labor force. For simplicity, we rule out depreciation.

If capitalists use optimal control in order to choose how much to consume and how much to save, and if they have logarithmic preferences over consumption streams, the capital stock will grow at the rate:

$$\frac{\dot{K}}{K} \equiv g_k = v - \beta,$$

where v is the profit rate and β is the capitalists' rate of time preference. By simple accounting,

$$v = (1 - \omega)\rho,$$

so that the accumulation rate is:

$$g_k = (1 - \omega)\rho - \beta \tag{1}$$

Further, constancy of the output/capital ratio ρ has the following implication:

$$\left(\frac{\dot{K}}{Y}\right) = 0 \iff \frac{\dot{K}Y - \dot{Y}K}{Y^2} = 0 \iff \frac{\dot{K}}{Y} - \frac{\dot{Y}}{Y}\frac{K}{Y} = 0 \iff \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y}$$

so that the accumulation rate also gives us the growth rate of real GDP. On the other hand, the production technology gives an employment rate equal to

$$e = L/N = \frac{\rho K}{x}\frac{1}{N} = \frac{Y}{xN}$$

If we take the logarithmic derivative of the employment rate with respect to time, we see that the growth rate of e satisfies

$$\frac{\dot{e}}{e} = g_k - (\gamma + n)$$

so that, using (1), we have the following dynamic equation for the employment rate:

$$\dot{e} = [(1-\omega)\rho - (\beta + \gamma + n)]\, e \tag{2}$$

Let us think a bit about this equation. The time change in the employment rate is decreasing in the labor share ω, which is consistent with standard descriptions of the labor market. The more labor becomes expensive, the lower demand for labor by firms. Further, as long as $\rho > \gamma + n + \beta$ —which in turn is required for a meaningful steady state, as we will see below— employment is always increasing when the labor share is zero: the employment rate acts then like a *prey* in a Lotka-Volterra biological system.[1]

Next, we turn to the dynamics of the labor share. By definition, $\omega \equiv w/x$, so that its growth rate will be equal to the difference between the growth rate of the real wage and the growth rate of labor productivity. Appealing to Phillips Curve considerations, Goodwin hypothesized that the growth rate of real wages increases with the employment rate: $\dot{w}/w = f(v), f(0) < 0, f'(v) > 0, \lim_{v \to 1} f(v) = \infty$. A linear approximation leads to[2]

$$\frac{\dot{w}}{w} = -\phi + \lambda e, \lambda > 0$$

Then, the dynamics of the labor share are

$$\dot{\omega} = [-\phi + \lambda e - \gamma]\omega \tag{3}$$

[1] Typically, the long-run value for ρ in the United States is 0.4. With a population growth rate in the neighborhood of 2% a year, a long-run growth rate of labor productivity of about 2% a year, and a discount rate of 1%, the story holds.

[2] We will see when looking at numerical simulations that a linear approximation creates problems in the dynamics, because the employment rate can cross its upper bound of 100%. A better assumption is $f(v) = -\phi + \frac{\sigma}{1-e}$.

The time change in the labor share increases in the employment rate, and for $e = 0$, $\dot{\omega} < 0$ always: the labor share acts like a *predator* in a biological system.

1.2 The Dynamical System

We now have a non-linear dynamical system in the state space (e, ω) that we are interested in studying. First, let's look at steady states. Obviously, if $e = 0 = \omega$, the system is in a steady state. However, this steady state is not interesting, so we rule it out. On the other hand, we have another steady state at

$$e_{ss} = \frac{\phi + \gamma}{\lambda} \tag{4}$$

$$\omega_{ss} = 1 - \frac{\beta + \gamma + n}{\rho} \tag{5}$$

Observe that the steady state value for the employment rate is independent of the labor share, and that the steady state value of the labor share is independent of the employment rate. Further, in order to have a steady state labor share between zero and one, we must impose $\rho > \beta + \gamma + n$. Similarly, for the employment rate to be bounded above by one, $0 < \phi + \gamma < \lambda$.

If we wanted to plot these steady state values (that is, isoclines) in a graph with e on the horizontal axis and ω on the vertical axis, we would have a vertical and a horizontal line respectively. The steady state is found at the intersection of these two lines.

An important implication of the steady state of the model is that an increase in workers' bargaining power λ has no effect on the long run income distribution, but only a negative effect on employment. Further, an increase in β, the discount rate of capitalists is unambiguously detrimental to the steady state labor share. In other words, workers have a lot to gain from a thriftier capitalist class in terms of their long run distributional position.

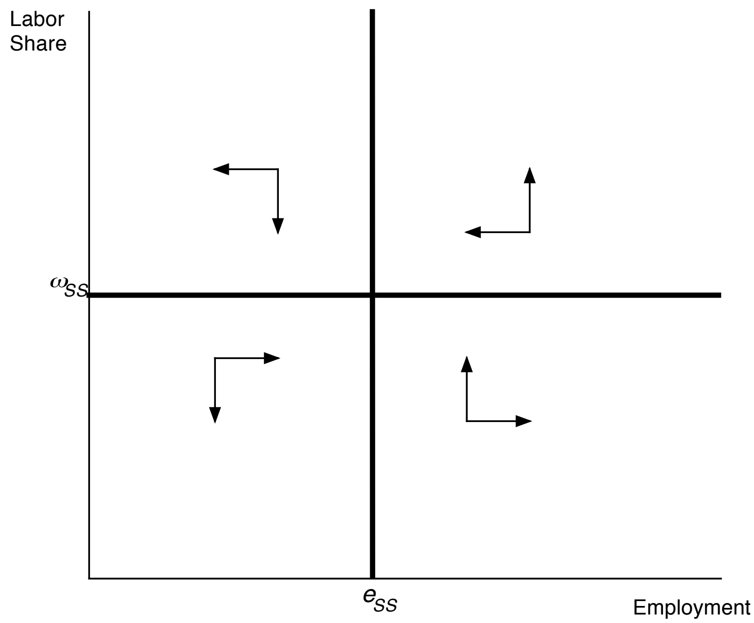

Figure 1: Phase Diagram of the Goodwin Model

2 Out of Steady–State Dynamics

2.1 Phase Diagram

Of particular interest is the dynamics of the system generated by the model. We can visualize the dynamics by drawing a *phase diagram*. We already know how the *isoclines* look like. Inspections of the dynamic equation for the labor share shows that the growth rate of ω increases with e out of the steady state. Hence, the dynamics of ω will be such that the labor share decreases when employment is below its steady state level, while it increases when employment is above its steady state level. We can draw arrows pointing down to the left of e_{ss} and up to the right of e_{ss}. On the other hand, the growth rate of employment decreases with the labor share out of the steady state. Hence, we can draw arrows pointing right below the ω_{ss} isocline, and left above it. As Figure 1 shows, we have a counterclockwise dynamical movement of the two variables around the steady state. Will they converge to it? In other words, is the Goodwin system stable? To answer this question, we need to linearize the system and study its Jacobian matrix.

2.2 Formal Stability Analysis

To characterize whether the labor share and the employment rate converte or not to their steady state values, let us linearize the system around its steady state position. The Jacobian matrix is:

$$J(e_{ss}, v_{ss}) = \left\{ \begin{array}{cc} 0 & -\rho\dfrac{\phi+\gamma}{\lambda} \\ \lambda\left(1-\dfrac{\beta+\gamma+n}{\rho}\right) & 0 \end{array} \right\} \tag{6}$$

with $Det(J) = (\phi+\gamma)(\rho-\beta-n-\gamma) > 0$, and $Tr(J) = 0$. In a 2D system like this one, we know that the sum of the real parts of the eigenvalues of the system is equal to the trace of the Jacobian. In this case, the trace is zero, so that the real parts of the eigenvalues are zero. However, eigenvalues can in general have imaginary, other than real parts. The determinant is positive, which means that the steady state is unstable. However, let us calculate the eigenvalues $\epsilon_{1,2}$ of the Jacobian matrix. These are found by setting

$$Det\left\{ \begin{array}{cc} -\epsilon & -\rho\frac{\phi+\gamma}{\lambda} \\ \lambda\left(1-\frac{\beta+\gamma+n}{\rho}\right) & -\epsilon \end{array} \right\} = 0$$

which yields the following *characteristic equation*:

$$\epsilon^2 + (\phi+\lambda)(\rho-\beta-\gamma-n) = 0$$

whose solutions are:

$$\epsilon_{1,2} = \sqrt{-(\phi+\lambda)(\rho-\beta-\gamma-n)} = \pm\sqrt{(\phi+\lambda)(\rho-\beta-\gamma-n)}i$$

where $i = \sqrt{-1}$ is the imaginary unit, which appears there since we are taking the square root of the negative number $-(\phi + \lambda)(\rho - \beta - \gamma - n)$.

Because the eigenvalues are purely imaginary, any dynamical trajectory starting off the steady state will cycle forever around it without ever reaching it. We say that any trajectory off the steady state is a *closed orbit*, neither diverging nor converging to the steady state.

Hence, the economy never gets to the steady state, but cycles around it forever, depending on initial conditions. In other words, distributional cycles (conflict) are *persistent* in the Goodwin model.

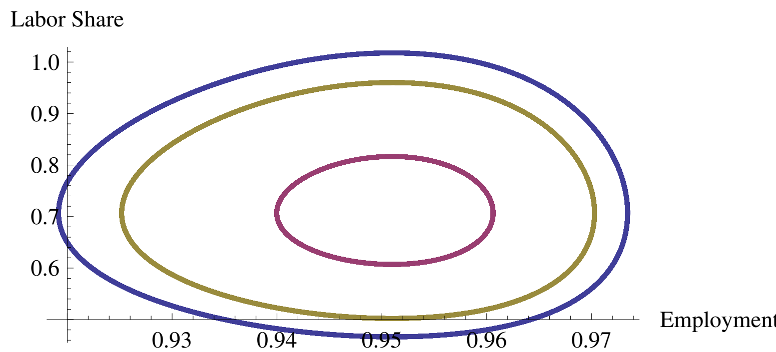

Figure 2: Closed orbits in the Goodwin Model for different initial conditions

From a mathematical standpoint, the reason why this is so has to do with the zero trace of the Jacobian. The economics is that there is no feedback from the labor share to its own growth rate, nor a feedback from the employment rate to its own growth rate. Inspection of equation (2) reveals that one possible reason can be found in the assumption of purely exogenous technical change. We will see that relaxing this assumption leads to some dramatic changes in the dynamics of distributional conflict.

3 Goodwin Cycles in the US (1963-2003)

Figure 3 plots annual values for the labor share and the employment rate in the United States between 1960 and 2003, and subtracts the top 10%, top 5% and top 1% from the

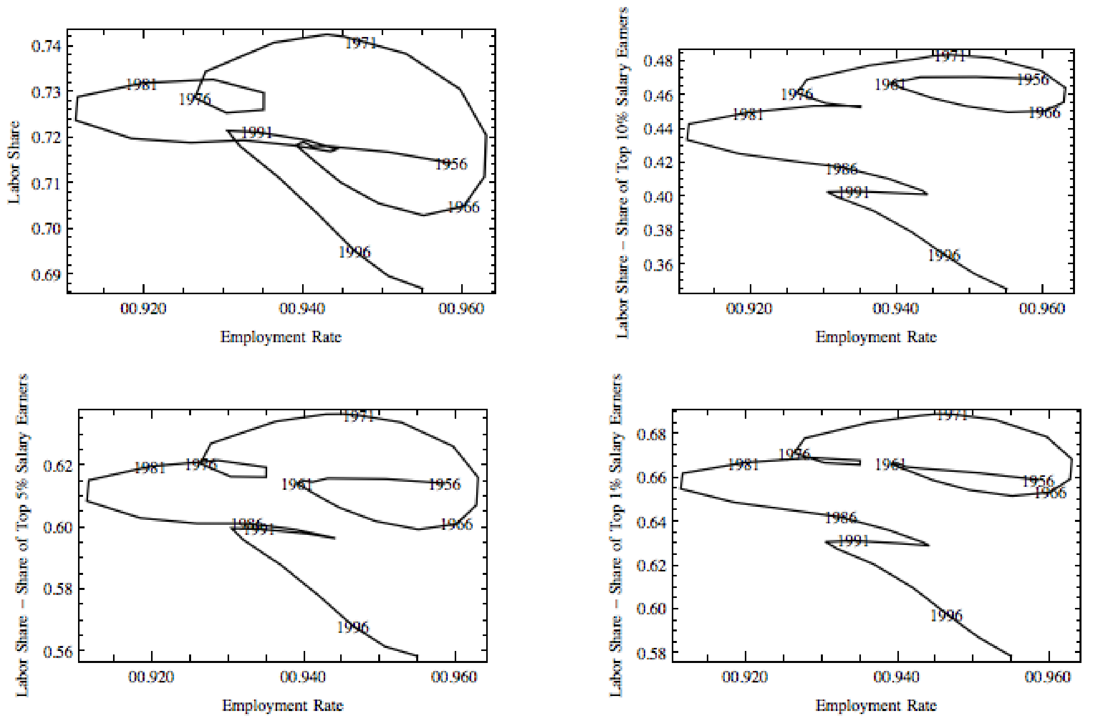

Figure 3: Employment–Distribution Cycles in the US (1963-2003). Source: BLS

labor share to control for the increase in wage inequality after 1970s. The counterclockwise movement is apparent, though cycles are never completed and they shift over time. Shifts occur because of institutional changes and other shocks.

References

Goodwin, Richard, 1967. 'A growth cycle', in: Carl Feinstein, editor, *Socialism, capitalism, and economic growth.* Cambridge, UK: Cambridge University Press.